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The Lebesgue Integral and Measure Theory

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Abstract: This article shows the aspects of the Lebesgue Integral and the Measure Theory, where the mathematical knowledge can be applied, how to do it and what led to the discovery of the Lebesgue Integration. The limitation of applying the theory will also be discussed. Understanding the field of mathematics would require the wholesome interest in doing calculations and solving the problems that may be difficult to solve using other methods.

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1 Introduction

The practical approach under which the knowledge of mathematics can be applied will rely on what field the knowledge is required. When the limits are given as a and b, for a given function, then the area under the curve can be worked out using the integral of the given function. When solving for the areas under the polynomial functions, it becomes different when compared with the more exotic functions. For a mathematician to get the area under the exotic function more of the theoretical approach has to be involved so as to land to the right answer. Lebesgue integral becomes necessary now when it comes to the calculation of the integrals that includes the broader classes of the functions. The application of the Lebesgue Integral makes it an easy way of solving the function since the values can be freely rearranged without interfering with the values of the integral
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2. Lebesgue Integration

The integral requires the knowledge on the use of the measurement that is rightly possible to associate each of the given set A that contains the real numbers. By having the idea on the size of the given portions that have the lengths of specific intervals gives a distinct expected answer when using the Lebesgue Integration (Bressoud, 2008). The extension of the use of the Lebesgue Integral makes the use of the theory of measurements. In the beginning, the Riemann integration was used in solving the given functions but later paved the way for the Lebesgue Integration, which provides the basics in solving the exotic functions.

When solving the integral functions, the consideration of the nature of the intervals has to be made. The two categories of the intervals that is, the finite and infinite intervals are mostly used in mathematics. The Lebesgue integral solves the functions directly as opposed to the Riemann. The increasing sequences that get to be bounded above are easily approached and solved through the Lebesgue Integration (Federer, 2014). There are several theorems that aid in the solving of the functions while applying the knowledge of Lebesgue shows the relevance of meeting certain conditions. For instance the theory below:
Suppose that *I*= (-∞, *H*] , where *H*ЄR. Supposing also that the function *f*: *I*→R satisfies the following conditions:

i. *F* Є *L*([A,B]) for every real number A ≤ B.
ii. There exists a given constant, M , such that $\int_A^B |f(x)|dx \leq M$ for every real number A ≤ B. Then *F* Є *L*(I), the limit $\int_A^B f(x)dx$ exists, and $\int_{-\infty}^{B} f(x)dx = \lim A \to -\infty \int_A^B f(x)dx$ (Federer, 2014)

3. Limitations of Lebesgue Integration

If the function is Riemann integral, then the Lebesgue Integration is possible. The statement only holds if the all the integrals are proper. The improper integrals cannot be said to be validly solved through the Lebesgue integration.
A function is thus said to be Lebesgue integrable if and only if the values used is absolute and that the function *f* is not in *L.*

4. Measure theorem

Lebesgue measure originated from the one of the French mathematician named Henri Lebesgue, who made it possible to have a common standard way that involves the assigning of the measures to the subsets that have n- dimensional. The measure theorem provides an analysis of the lengths, area, and volumes of the Euclidean spaces. The notions of the measurements are generalized in this field, and the practical answers are thus made easy to solve. The measure is taken as any function that is defined on any given subsets that tend to satisfy the given list of properties. Measure theory is thus one of the branches of the real analysis. The theory is useful when investigating the

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measurable functions and integrals of the given functions (Bartle, 2001). As a characteristic of the Lebesgue measure, the use of the outer measures makes it easy to translate the values in the given exotic functions. The subsets of the real numbers are taken as the conditions when solving the functions. Through the mathematical knowledge, the existence of the given sets that do not meet the Lebesgue measure conditions provides a consequence of a set of certain set-theoretical axioms. These can be interpreted to be independent of given number of the conventional systems that relate to the axioms of the set theory.

5. The application of the Lebesgue Integration and Measure

Many of the times the mathematicians make use of the knowledge gained in solving the Lebesgue integrals by solving the partial differential equations (PDEs) when the derivatives are weak. Two steps are necessary involved that shows a weak solution exists while the proving of the presence of regularity result is done (Stein & Shakarchi, 2009). Another case is when the one is solving the Fourier transformations. The presence of the various numbers of the results on the convergence of the Fourier transformations invites the knowledge of the Lebesgue on proving them. Lebesgue integration put the Fourier transformations on the solid thus easing out the way to solve the problems.

The other application that involves the use of the Lebesgue is on the probability. The modern view of the probability applies the measure theory (Halmos, 2013). The probability space is easily understood since once can make the sense that the things said are real.

Lebesgue integral is necessary for mathematics as one is allowed to deal the functions in an excellent and elegant way as compared to the Riemann integral. In mathematics, things such as the function space are easily solvable since there is a rigorous foundation. In most of the mathematical physics, the Lebesgue integration and measure theory forms foundations to solve the physics related equations.

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